CDI-II Prática F5 5/4/21

— 4 —

1- Trivial

2- $V(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$
\begin{tikzcd}
(x,y,z) \arrow[r, mapsto, "r"] \arrow[rr, bend right, "V = f \circ r"'] & \sqrt{x^2+y^2+z^2} \arrow[r, mapsto, "f"] & \dfrac{1}{\sqrt{x^2+y^2+z^2}}
\end{tikzcd}
$$

$$\mathbb{R}^3 \longrightarrow \mathbb{R} \longrightarrow \mathbb{R}$$

$$r \longmapsto f(r) = \frac{1}{r}$$

$$(x,y,z) \longmapsto r(x,y,z) \longmapsto f(r(x,y,z))$$

$$V(x,y,t) = f(r(x,y,z))$$

$$\frac{\partial V}{\partial x}(x,y,z) = \underbrace{f'(r(x,y,z))}\,\underbrace{\frac{\partial r}{\partial x}(x,y,z)}$$

$$\frac{\partial^2 V}{\partial x^2}(x,y,t) = f''(r(x,y,t))\,\frac{\partial r}{\partial x}(x,y,t)\,\frac{\partial r}{\partial x}(x,y,t)$$

$$+ f'(r(x,y,t))\,\frac{\partial^2 r}{\partial x^2}(x,y,t)$$

$$r(x,y,t) = \sqrt{x^2+y^2+t^2} = (x^2+y^2+t^2)^{1/2}$$

$$f(r) = \frac{1}{r} \qquad f'(r) = \frac{df}{dr}(r)$$

etc. Contas

3- $w(x,y) = f(y-x, x+y) = f(u(x,y), v(x,y))$

$f : \mathbb{R}^2 \to \mathbb{R}, \ C^2.$

$u = y - x$

$v = x + y$

$$\frac{\partial w}{\partial x}(x,y) = \frac{\partial f}{\partial u}\big(u(x,y), v(x,y)\big)\frac{\partial u}{\partial x}(x,y) +$$

$$+ \frac{\partial f}{\partial v}\big(u(x,y), v(x,y)\big)\frac{\partial v}{\partial x}(x,y)$$

$$= -\frac{\partial f}{\partial u}\big(u(x,y), v(x,y)\big)$$

$$+ \frac{\partial f}{\partial v}\big(u(x,y), v(x,y)\big)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v}\right)$$

$$\frac{\partial^2 w}{\partial x^2} = -\left[\frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial x} + \frac{\partial^2 f}{\partial v \partial u}\frac{\partial v}{\partial x}\right] +$$

$$\left[\frac{\partial^2 f}{\partial u \partial v}\frac{\partial u}{\partial x} + \frac{\partial^2 f}{\partial v^2}\frac{\partial v}{\partial x}\right]$$

$$= -\left(-\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v \partial u}\right) +$$

$$+\left(-\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2}\right)$$

$$\underline{(x,y)} \longmapsto \big(u(x,y), v(x,y)\big) \longmapsto f\big(y-x, x+y\big)$$

$$(\underset{u}{y-x},\ \underset{v}{x+y})$$

$$w(x,y) = f\Big(\underbrace{u(x,y)}_{y-x}, \underbrace{v(x,y)}_{x+y}\Big)$$

$$\frac{\partial w}{\partial y}(x,y) = \frac{\partial f}{\partial u}\overset{=1}{\frac{\partial u}{\partial y}} + \frac{\partial f}{\partial v}\overset{=1}{\frac{\partial v}{\partial y}}$$

$\big(u(x,y), v(x,y)\big)$

(x,y)

$$\frac{\partial w}{\partial y}(x,y) = \frac{\partial f}{\partial u}\Big(u(x,y), v(x,y)\Big) + \frac{\partial f}{\partial v}\Big(u(x,y), v(x,y)\Big)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 f}{\partial u^2}\overset{=1}{\frac{\partial u}{\partial y}} + \frac{\partial^2 f}{\partial v \partial u}\overset{=1}{\frac{\partial v}{\partial y}} +$$

$$+ \frac{\partial^2 f}{\partial u \partial v}\overset{=1}{\frac{\partial u}{\partial y}} + \frac{\partial^2 f}{\partial v^2}\overset{=1}{\frac{\partial v}{\partial y}}$$

$$\frac{\partial^2 w}{\partial y^2} = \cancel{\frac{\partial^2 f}{\partial u^2}} + 2\frac{\partial^2 f}{\partial u \partial v} + \cancel{\frac{\partial^2 f}{\partial v^2}}$$

$$-\frac{\partial^2 w}{\partial x^2} = -\cancel{\frac{\partial^2 f}{\partial u^2}} + 2\frac{\partial^2 f}{\partial u \partial v} - \cancel{\frac{\partial^2 f}{\partial v^2}}$$

$$\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} = 4\frac{\partial^2 f}{\partial u \partial v}$$

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4 - $Hf(a)_{n \times n}$

$$Tr(Hf(a)) = \lambda_1 + \dots + \lambda_n$$

$$\det(Hf(a)) = \lambda_1 \times \dots \times \lambda_n$$

$n = 2 \rightarrow$ 2 eq. 2 incógnitas

$n > 2 \rightarrow$ sist. ind.

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4-e)

$$\begin{cases} \frac{\partial f}{\partial x} = z - 2x = 0 \\ \frac{\partial f}{\partial y} = -2y = 0 \\ \frac{\partial f}{\partial z} = x = 0 \end{cases}$$

$(0,0,0)$ unico p^{to} critico de f.

$$Hf(0,0,0) = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\det\left(Hf(0,0,0) - \lambda I \right) = 0$$

$$\det \begin{bmatrix} -2-\lambda & 0 & 1 \\ 0 & -2-\lambda & 0 \\ 1 & 0 & -\lambda \end{bmatrix} = 0$$

$$(-2-\lambda) \det \begin{bmatrix} -2-\lambda & 1 \\ 1 & -\lambda \end{bmatrix} = 0$$

$$(-2-\lambda)\left[-\lambda(-2-\lambda) - 1\right] = 0$$

$$-2-\lambda = 0 \quad \vee \quad -\lambda(-2-\lambda) - 1 = 0$$

$$\lambda = -2 \quad \vee \quad 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda = -2 \quad \vee \quad \lambda^2 + 2\lambda - 1 = 0$$

$$\lambda = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

$$\underbrace{\lambda = -2 \ \vee \ \lambda = -1-\sqrt{2}}_{<0} \ \vee \ \underbrace{\lambda = -1+\sqrt{2}}_{>0}$$

4-g) $f(x,y) = x^3 - y^2$

$$\begin{cases} 3x^2 = 0 \\ -2y = 0 \end{cases} \iff \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$Hf(x,y) = \begin{bmatrix} 6x & 0 \\ 0 & -2 \end{bmatrix}$$

$$Hf(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}$$

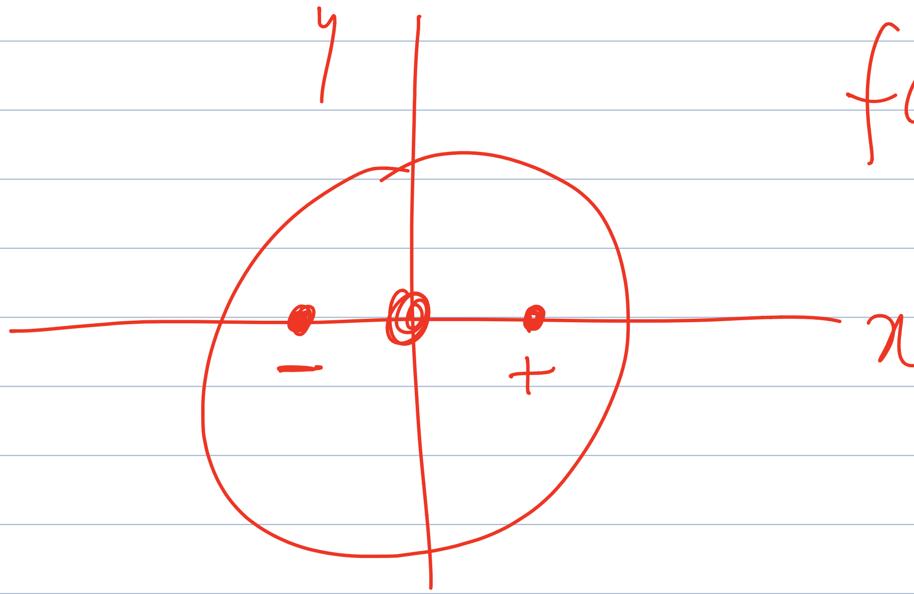
$\lambda_1 = 0$

$\lambda_2 = -2$

Análise local "perto" de $(0,0)$

$f(0,0) = 0$

$f(x,y) - f(0,0) =$
$= f(x,y)$



$f(0,0)=0$

$$f(x,y) = x^3 - y^2$$

$$f(x,0) = x^3 \begin{cases} <0 & \text{se } x<0 \\ >0 & \text{se } x>0 \end{cases}$$

$(0,0)$ não é ponto de extremo de f.